

Homework №04

Name:.....

Exercice 01:

A farmer owns a perfectly square field and wants to estimate its area. When he measures one side of his field, he knows (confirmed by a statistician friend) that the measurement error follows a normal distribution with zero mean and standard deviation σ . He decides to take two measurements, which he assumes to be independent, and seeks which estimator to choose from the two measurements.

Let X be a random variable corresponding to a length obtained after a measurement. We have $X \sim N(\mu; \sigma)$, where μ is the true length of the side and σ is the measurement error. Thus, the value θ he seeks to estimate is μ^2 . Since the farmer only wants to make two measurements, he has only a 2-sample for the construction of an estimator.

He has three ideas for estimators based on these two measurements:

- $T_1 = X_1 \times X_2$;
- $T_2 = \frac{1}{2} (X_1^2 + X_2^2)$;
- $T_3 = \left(\frac{X_1 + X_2}{2} \right)^2$.

For each estimator, calculate its bias and mean squared error.

Hint: $E(X^2) = V(X^2) + E(X)^2$ and $V(X^2) = 2(\sigma^4) + 2\mu\sigma^2$

2. Based on your results, which estimator among the three would you recommend to the farmer? Justify your choice.

Exercice 02:

Calculate the Fourier transforms of the following discrete signals :

a- $x(n) = u(n) - u(n - 6)$
b- $x(n) = 2^n u(-n)$

