



## Serie N° 8

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### Exercice N°1 :

Consider a sample of size  $n$ ,  $X_1, X_2, \dots, X_n$ , with mean and variance  $\sigma^2$ .

- 1- Provide the method of moments estimator for  $\mu^2$ .
- 2- Calculate its bias.
- 3- Determining  $k$  such that  $\bar{X}^2 - kS_{cor}^2$  is an unbiased estimator of  $\mu^2$ .

### Exercice 02 :

Let  $X_1, X_2, \dots, X_n$  be a sample with mean  $\mu$ . We are interested in two estimators of  $\mu$ .

$$T_1 = \frac{X_1 + X_2}{2} \quad \text{and} \quad T_2 = \frac{1}{n} \sum_{i=1}^n X_i - \frac{X_1 + X_2}{2}$$

Determine which of these two estimators is better in terms of mean squared error.

### Exercice 03:

Let  $X_1, X_2, \dots, X_n$  be a random sample from a distribution whose exact form is unknown, but we know that the probability density function is of the form  $f(x) = \theta x^{\theta-1}$  for  $0 < x < 1$ , where  $\theta$  is an unknown parameter.

1. Give the method of moments estimator for  $\theta$ .
2. Calculate the bias of this estimator.
3. Calculate the variance of this estimator.
4. Propose an estimator based on the method of moments.
5. Propose an estimator based on the maximum likelihood method.