Serie N°5

Exercice N°1 : Uniform phase sinusoidal process

We consider the random process defined by $X(t) = acos(2\pi t + \theta)$; where a and f_0 are constants and θ is a random variable equally distributed over (0, 2π).

1- For fixed t, find the probability density of the random variable X(t).
2- Calculate $E[X(t)]$ et $\overline{X(t)}$.
3- Calculate $E[X(t)X(t+\tau)]$ et $\overline{X(t)X(t+\tau)}$.

Exercice N°2

Given the following random signals, calculate the statistical average as well as the root mean square value and the autocorrelation function of each signal. What are stationary and ergodic signals?

a- X(t)=A
A is a random variable distributed between 0 and 1.

b- $X(t)=\cos(2\pi f_0 t+\varphi)$

f_0 is constant, φ is a random variable equally distributed over (0, φ_{max}). This process is generally non-stationary. Are there values of φ_{max} for which the process is stationary?

c- $X(t)=A\cos(2\pi f_0 t+\varphi)$

f_0 is constant, A and φ are independent random variables on and φ is uniformly distributed between 0 and 2π; A is uniformly distributed between 0 and 1.

Is this signal ergodic?

Exercice N°3

We consider a random process

$X(t)=A\cos(2\pi f_0 t+\varphi)$

f_0 is constant, A and φ are independent random variables. We assume that A follows a reduced centered normal distribution and that φ is uniformly distributed over [0, 2π].

we recall that

$$\cos(a)\cos(b) = \frac{1}{2}(cos(a+b) + \cos(a-b))$$

$$\cos(a)\sin(b) = \frac{1}{2}(sin(a+b) - \sin(a-b))$$

1- Is X(t) stationary ?
2- Is X(t) with an ergodic mean?
3- We now assume that φ =0. Are the previous statements still true?